Sample Question Paper

Mathematics

Class – 10+2

Time Allowed : 3 Hours **Maximum Marks : 80**

<u>Instructions</u> :

1) All questions are compulsory.
2) Question paper consists of 18 questions divided into 4 sections A, B, C and D.
3) Section A comprises of 1 question of 20 multiple choice type questions of 1 mark each.
4) Section B comprises of 7 questions of 2 marks each.
5) Section C comprises of 7 questions of 4 marks each.
6) Section D comprises of 3 questions of 6 marks each.
7) An internal choice is provided in 3 questions of Section C and D each. You have to attempt only one of the alternatives in all such cases.
8) Use of calculator is not allowed.

Section – A

Q1 Choose the correct options in the following questions :

(i) Relation $R = \{(x, y) : x < y^2 \text{ where } x, y \in \mathbb{R}\}$ is — 1

(a)Reflexive but not symmetric (b)Symmetric and transitive but not Reflexive
(c)Reflexive and Symmetric (d)Neither reflexive nor symmetric nor transitive

(ii) Range of function $\cos^{-1}$ is : — 1

(a)$[0, \pi] - \left\{\frac{\pi}{2}\right\}$ (b)$(0, \pi)$ (c)$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \{0\}$ (d)$[0, \pi]$

(iii) Principal value of $\sin^{-1}\left(\frac{1}{2}\right)$ is — 1

(a)$\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

(iv) If A is a square matrix of order 2×2 and $|A| = 5$ then $|Adj.(A)|$ is — 1

(a)25 (b)125 (c)5 (d)10

(v) If $\begin{bmatrix} x-2y & 0 \\ 5 & x \end{bmatrix} = \begin{bmatrix} -5 & 0 \\ 5 & 3 \end{bmatrix}$, then y is equal to:- — 1

(a)1 (b)3 (c)2 (d)4

(vi) If the order of the matrix A is 3×2 then the order of the matrix $(A')'$ is : — 1

(a)2×3 (b) 3×2 (c) 2×2 (d) 3×3

(vii) If $f(x) = \begin{cases} \frac{\sin 8x}{5x}, & x \neq 0 \\ m+1, & x = 0 \end{cases}$ is continuous at $x = 0$ then value of m is — 1

(a)$\frac{5}{8}$ (b)$\frac{8}{5}$ (c)$\frac{3}{5}$ (d)$\frac{5}{3}$

(viii) If $y = e^{\log x}$ then $\frac{dy}{dx}$ is — 1

(a)$\log x - x$ (b)$xe^{\log x}$ (c)1 (d)$e^{\log x} \log x$

(ix) If $y = \tan x$ then, at $x = \frac{\pi}{4}$, $\frac{dy}{dx}$ is equal to : — 1

(a)1 (b)$\sqrt{2}$ (c)2 (d)4

(x) Radius of a circle is increasing at the rate of $2\ m/s$. Rate of change of its circumference is : — 1

(a)$4\pi\ m/s$ (b)$2\ m/s$ (c)$2\pi\ m/s$ (d)$4\ m/s$

(xi) $\int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$ is equal to — 1

(a)$\frac{\pi}{4}$ (b)$\frac{\pi}{6}$ (c)$\frac{\pi}{12}$ (d)$\frac{\pi}{2}$

(xii) $\int_0^1 \frac{dx}{1+x^2}$ is equal to : — 1

(a)$\frac{\pi}{2}$ (b)$\frac{\pi}{4}$ (c)$\frac{\pi}{3}$ (d)$\frac{\pi}{6}$

(xiii) Order of differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3y = 0$ is: — 1

(a)3 (b) 2 (c)1 (d)0

(xiv) If $\vec{a}$ is a non-zero vector then $|\vec{a} \times \vec{a}|$ is equal to — 1

(a)$|\vec{a}|$ (b)$|\vec{a}|^2$ (c)1 (d)0

(xv) Name of the inequality $|\vec{a}.\vec{b}| \le |\vec{a}||\vec{b}|$ is : 1

(a)Cauchy-Schwartz Inequality (b)Triangle Inequality
(c)Rolle's Inequality (d)Lagrange's Inequality

(xvi) Direction ratios of the straight line $\vec{r} = 2\hat{\imath} - 3\hat{\jmath} + \hat{k} + m(9\hat{\imath} - 2\hat{\jmath} + 5\hat{k})$ are 1

(a)$< 2, -3, 1 >$ (b)$< 9, 2, 5 >$ (c)$< -2, 3, -1 >$ (d)$< 9, -2, 5 >$

(xvii) Vector equation of the line $\frac{x-5}{-4} = \frac{y-3}{5} = \frac{z+3}{-8}$ is 1

(a)$\vec{r} = 4\hat{\imath} - 5\hat{\jmath} - 8\hat{k} + \mu(5\hat{\imath} + 3\hat{\jmath} - 3\hat{k})$ (b)$\vec{r} = -4\hat{\imath} + 5\hat{\jmath} + 8\hat{k} + \mu(5\hat{\imath} + 3\hat{\jmath} - 3\hat{k})$
(c)$\vec{r} = 5\hat{\imath} + 3\hat{\jmath} - 3\hat{k} + \mu(4\hat{\imath} - 5\hat{\jmath} - 8\hat{k})$ (d)$\vec{r} = 5\hat{\imath} + 3\hat{\jmath} - 3\hat{k} + \mu(-4\hat{\imath} + 5\hat{\jmath} - 8\hat{k})$

(xviii) Objective function of a linear programming problem is : 1

(a)Always quadratic (b)Always linear
(c)May be linear or quadratic depending on the problem (d)May be cubic some times

(xix) If $P(A) = \frac{1}{2}$, $P(B) = \frac{3}{8}$ and $P(A \cap B) = \frac{1}{5}$ then $P(A|B)$ is equal to : 1

(a)$\frac{2}{5}$ (b) $\frac{8}{15}$ (c) $\frac{2}{3}$ (d) $\frac{5}{8}$

(xx) Ram and Rahim are contesting for two vacancies in a company. Probability of selection of Ram is $\frac{7}{9}$ and that of Rahim is $\frac{4}{7}$. What is the probability that both will be selected ? 1

(a)$\frac{61}{63}$ (b)$\frac{4}{9}$ (c)$\frac{8}{9}$ (d)$\frac{11}{16}$

Section – B

Q2 If $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}$ then verify that $(AB)' = B'A'$ 2

Q3 Find $\frac{d}{dx}(x^x)$ and evaluate whether this result is true $\forall\, x \in \mathbb{R}$. 2

Q4 Evaluate $\int \frac{2x-3}{x^2+1} dx$. 2

Q5 Using integration, find the area bounded by the circle whose centre is at origin and radius is 4 units . 2

Q6 The volume of spherical balloon is increasing at the rate of 25 c.c./s . Find the rate of change of its surface area at the instant when its radius is 5cm. 2

Q7 Find the points on the curve $y = 2x^3 - 3x^2 - 12x + 15$ where rate of change of dependent variable with respect to the independent variable vanishes. What do we call these type of points ? 2

Q8 Find the value of p if the vectors $p\hat{\imath} - 8\hat{\jmath} + 5\hat{k}$ and $5\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ are perpendicular to each other. 2

Section – C

Q9 Prove that the function, $f: \mathbb{R} \to \mathbb{R}, f(x) = \frac{5x+3}{4}$ is one-one and onto. 4

Q10(a) Using determinants, find the value of k if the area of the triangle formed by the points $(-3, 6), (-4, 4)$ and $(k, -2)$ is 12 sq. units. 2

(b) If $X = \begin{bmatrix} 3 & 4 \\ 2 & -1 \end{bmatrix}$ and $2X - Y = \begin{bmatrix} 5 & 10 \\ 3 & -5 \end{bmatrix}$ then find the matrix Y. 2

Q11 If $u = x^y$, $v = y^x$ and quantity y remains 3 times the quantity x then find that amongst quantities u and v, which changes more rapidly with respect to quantity x when $x = 1$. (Take $\log_e 3 = 1.09$) 4

OR

If $x = \frac{1-t^2}{1+t^2}$, $y = \frac{2t}{1+t^2}$ then prove that $\frac{dy}{dx} + \frac{x}{y} = 0$.

Q12 Evaluate $\int \frac{3x+2}{(x^2+1)(x-2)} dx$ 4

Q13 Solve the following linear programming problem graphically : 4
Maximize and minimize $Z = 4x + 2y - 7$ subject to the constraints

$$x + 3y \le 60, x + y \ge 10, x - y \le 0, x \ge 0, y \ge 0$$

Q14 Solve : $x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x.$ 4

OR

Solve : $x^2dy-\left(3x^2+xy+y^2\right)dx=0$; given that $y=1$ when $x=1$. 4

Q15 Bag I contains 7 red and 5 white balls. Bag II contains 3 red and 4 white balls. A bag is selected at random and a ball is drawn from it. Detect that which ball has more chances of being drawn red or white ? 4

OR

A laboratory blood test is 99% effective in detecting a certain disease when it is in fact present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then, with the probability 0.005 , the test will imply he has the disease). If 0.1% of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive ? 4

Section – D

Q16 Ajay, Sameer and Meenal have Rs.20/- each and some footballs, basketballs and volleyballs in their shops. In a week, Ajay sold 3 footballs and a volleyball but he bought 2 basketballs for his shop and he has Rs.35/- now. In same duration, Sameer sold 2 basketballs and 2 volleyballs but he bought a football for his shop and he has Rs. 95/- now. Similarly, Meenal sold 2 footballs and a basketball but she bought 3 volleyballs for her shop and she has Rs.15/- now. Find the cost of a football, a basketball and a volleyball by the help of matrices. 6

OR

(a) Express $\begin{bmatrix} 5 & 1 \\ 3 & 7 \end{bmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix. 3

(b) If $A=\begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$ and $f(x)=x^2-10x+13$ then show that $f(A)=O$ and using this result find A^{-1}. 3

Q17(a) Prove that for any two non-zero vectors $\vec{a}$ and $\vec{b}$, $|\vec{a}+\vec{b}|\le|\vec{a}|+|\vec{b}|$. Also write the name of this inequality. 4

(b) Adjacent sides of a parallelogram are given by $6\hat{\imath}-\hat{\jmath}+5\hat{k}$ and $\hat{\imath}+5\hat{\jmath}-2\hat{k}$. Find the area of parallelogram. 2

OR

(a) Find the shortest distance between the following pairs of lines : 4

$$\vec{r}=\hat{i}-4\hat{j}+5\hat{k}+\mu\left(5\hat{i}+9\hat{j}+\hat{k}\right)\ \&\ \vec{r}=2\hat{i}+8\hat{j}-6\hat{k}+\lambda\left(3\hat{i}-2\hat{j}+\hat{k}\right)$$

(b) Find the angle between the lines 2

$$\vec{r}=3\hat{i}+8\hat{j}+3\hat{k}+\mu\left(\hat{i}+2\hat{j}-\hat{k}\right)\ \&\ \vec{r}=-3\hat{i}+9\hat{j}-\hat{k}+\lambda\left(5\hat{i}+3\hat{j}+4\hat{k}\right)$$

Q18 Find the height of the right circular cone of maximum volume, which is inscribed in a sphere of radius 12cm. 6

OR

Evaluate $\int\frac{x^2}{x^4+1}dx$ 6